**EECE 290**

**Final Exam, May 13, 2015**

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**A.** Determine *vO*(*t*), assuming *vSRC* = 0.1*u*(*t*) V.

**Solution:** *vO*(*t*) = -*CRdvSRC*/*dt* = -10×0.1*δ*(*t*) = -*δ*(*t*) V

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**B.** Determine the rms value of the periodic function

whose first period is shown.

**Solution:** The area under the square of the first half cycle

Is 1 + 4 + 1 = 6. The mean is 2 and the rms value is .

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**C.** Convolve *tu*(*t*) with *tu*(*t*).

**Solution:** .

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**D.** Derive the Fourier transform of *f*(*t*).

**Solution:** *f*(*t*) = *u*(-*t*) + *u*(*t* – 2); *F*(*jω*) =

 +

.

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**1.** *vSRC*1(*t*) and *vSRC*2(*t*) are the periodic

functions shown. Determine

*τ* so that *vO*(*t*) is

periodic, assuming

*A* = 16 V.

1. 1 s
2. 0.2 s
3. 0.5 s
4. 1.25 s
5. 2.5 s

**Solution:** For *vO*(*t*) to be periodic, the average current through the capacitor must be zero. This requires that , where *VSRC*1 and *VSRC*2 are the respective dc components, that is, *VSRC*1 = 5 V and *VSRC*2 = -*Aτ/2.* This gives: , or *τ =* 20/*A* s.

**Version 1:** *A* = 16; *τ =* 20/16 = 1.25 s

**Version 2:** *A* = 20; *τ =* 20/20 = 1 s

**Version 3:** *A* = 40; *τ =* 20/40 = 0.5 s

**Version 4:** *A* = 80; *τ =* 20/80 = 0.25 s

**Version 5:** *A* = 100; *τ =* 20/100 = 0.2 s.

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**2.** The full-wave rectified waveform in (a) has

, *n* = 1, 2, 3,… Determine *C*1*b* for the modified full-wave rectified waveform in (b).

1. 0.042
2. 0.085
3. -0.085
4. -0.042
5. 0.064

**Solution:** *C*1*a* = ; when shifted to the right or left by half a period, *C*1*a* is multiplied by ; when negated, the sign of *C*1*b* remains positive. The magnitude of *C*1*b* is 0.4 times that of *C*1*a*. It follows that *C*1*b* =  = 0.085.

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**3.** The source delivers a complex power of *j*8 kVA, with **VSRC** = 100∠30° V rms. Determine *C*, assuming *ω* = 1 krad/s.

1. 0.6 mF
2. 0.4 mF
3. 0.2 mF
4. 0.5 mF
5. 0.3 mF

**Solution:** |**S**| = |**VSRC**|2; *ωC* = 1 – = 1 – 0.1*S*, where *S* is the magnitude of the complex power in kVA, or *C* = (1 – 0.1*S*) mF.

**Version 1:** *S* = 8; *C* = (1 – 0.8) = 0.2 mF

**Version 2:** *S* = 7; *C* = (1 – 0.7) = 0.3 mF

**Version 3:** *S* = 6; *C* = (1 – 0.6) = 0.4 mF

**Version 4:** *S* = 5; *C* = (1 – 0.5) = 0.5 mF

**Version 5:** *S* = 4; *C* = (1 – 0.4) = 0.6 mF.

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**4.** The switch is closed at *t* = 0, with initial voltages on *C*1

and *C*2, and with *q*4(0-) = 4 C. Initial voltage on *C*3 is not shown. Determine the charge through the switch when it closes.

1. 12 C
2. 6 C
3. 9 C
4. 15 C
5. 10 C

**Solution:** *Ceq* = 2/3 + 4/3 = 2 F. *Veq* = 6 V. Hence, *qeq* = 6×2 = 12 C, which is the charge that passes through the switch.

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**5.** In the preceding problem, determine the final value of *v*4.

1. -0.5 V
2. 1 V
3. 0.5 V
4. -1.5 V
5. -1V

**Solution:** *Ceq*3,4 = 4/3 F, *qeq*3,4 = 6×4/3 = 8 C. This charge flows through *C*3 and *C*4 CCW. The final charge on *C*4 is 4 – 8 = -4 C, so the final voltage on *C*4 is -1 V.

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**6.** The initial currents in *L*1 and *L*4 are shown, those in *L*2

and *L*3 are not shown. Determine *i*3(0+), assuming

*VSRC* = 3*δ*(*t*) V.

1. 4 A
2. 6 A
3. 3 A
4. 7 A
5. 5 A

**Solution:** The circuit is equivalent to a 1 H inductor in series with a 0.5 H inductor, with the impulse applied across the series combination. Considering the inductors to be uncharged, an impulse 2*Kδ*(*t*)/3 appears across each of the 2 H inductors, resulting in a current *K*/3 A through each inductor, that adds to the initial current in the inductor. From KCL, *i*3(0-) = 2 A. It follows that *i*3(0+) = (*K*/3 + 2) A.

**Version 1:** *K* = 3; *i*3(0+) = (*K*/3 + 2) = 3 A

**Version 2:** *K* = 6; *i*3(0+) = (*K*/3 + 2) = 4 A

**Version 3:** *K* = 9; *i*3(0+) = (*K*/3 + 2) = 5 A

**Version 4:** *K* = 12; *i*3(0+) = (*K*/3 + 2) = 6 A

**Version 5:** *K* = 15; *i*3(0+) = (*K*/3 + 2) = 7 A.

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**7.** Determine *vO*(*t*), assuming an initial voltage

of 2 V on *C*1 and an initial voltage *V*20 = 2 V

on *C*2.

1.  V
2.  V
3.  V
4.  V
5.  V

**Solution:** The impulse passes through *C*1, depositing a charge of 4 μC, which reduces the voltage across *C*1 to zero. Hence, *vO*(0+) = *V*20; *vO*(∞) = 0; and *τ* = 3×8/6 = 4 ms. It follows that  V, *t* is in ms.

**Version 1:** *V*20 = 2 V;  V, *t* is in ms

**Version 2:** *V*20 = 3 V;  V, *t* is in ms

**Version 3:** *V*20 = 4 V;  V, *t* is in ms

**Version 4:** *V*20 = 5 V;  V, *t* is in ms

**Version 5:** *V*20 = 6 V;  V, *t* is in ms.

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**8.** Determine *vO*(*t*), assuming *iSRC* = 2*u*(*t*) A and no

initial energy storage.

1.  V, *t* is in s
2.  V, *t* is in s
3.  V, *t* is in s
4.  V, *t* is in s
5.  V, *t* is in s

**Solution:** From the T-equivalent circuit, the step current

*ISRC* produces a voltage impulse *Kδ*(*t*) between the middle

and lower nodes that instantaneously changes the currents in the 4 H and 6 H inductors. Since the flux linkage is the same, the currents in these inductors are inversely proportional

to the inductances, so that *i*2(0+) = *ISRC*×4/(6 + 4) = 0.4*ISRC*; hence, *v*O(0+) = 2*ISRC*; *vO*(∞) = 0; and *τ* = 10/5 = 2 s. It follows that  V, *t* is in s. Alternatively, *vO* = *MdiSRC*/*dt* = *MISRCδ*(*t*). hence, *i*2(0+) = (*M/L2)ISRC* = 0.4*ISRC*; *v*O(0+) = 2*ISRC*, as before.

**Version 1:** *ISRC* = 2 A;  V, *t* is in s

**Version 2:** *ISRC* = 3 A;  V, *t* is in s

**Version 3:** *ISRC* = 4 A;  V, *t* is in s

**Version 4:** *ISRC* = 6 A;  V, *t* is in s

**Version 5:** *ISRC* = 8 A;  V, *t* is in s.

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**9.** Given . Determine *f*(0+).

1. 0
2. infinite
3. 1
4. 2
5. Initial-value theorem does not apply

**Solution:** Multiplying by *s* and letting *s* → ∞, gives *f*(0+) = 0. Check: the PFE is: ; ; .

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**10.** The transfer function of a circuit is . If the steady-state output of the circuit is: *vO*(*t*) = 4 + 10cos(100*t* + 45°) V, determine the input *vI*(*t*).

1.  V
2.  V
3.  V
4.  V
5.  V

**Solution:** . Under dc conditions, *ω* = 0, so that *VIDC* = . For *ω* = 100 rad/s, ; it follows that *vI*(*t*) = 0.8 + .

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**11.** Given *v*(*t*) = 2sinc(2*t*) V. Determine the energy dissipated in a 2 Ω resistor over all time.

1. 7.07 J
2. 3.14 J
3. 1.07 J
4. 3.53 J
5. 6.28 J

**Solution:** FT of *v*(*t*)  is  or, FT of  is . Hence, FT of  is . The energy dissipated in a 1 Ω resistor is ×4 = =  J. The energy dissipated in a 2 Ω resistor is *W* =  J.

**Version 1:** *Vm* = 2 V; *W* =  = 3.14 J

**Version 2:** *Vm* = 3 V; *W* =  = 7.07 J

**Version 3:** *Vm* = 4 V; *W* =  = 12.57 J

**Version 4:** *Vm* = 5 V; *W* =  = 19.63 J

**Version 5:** *Vm* = 6 V; *W* =  = 28.27 J.

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**12.** The two-port circuit is described by its *g* parameters as:

*g*11 = (1 – *j*) S, *g*12 = (-1 – *j*), *g*21 = (1 + *j*), *g*22 = (1 + *j*) Ω.

Determine the complex power delivered by the source, assuming that **VSRC** = 2∠45° V rms. The *g*-parameter equations are: **I1** = *g*11**V1** + *g*12**I2**; **V2** = *g*21**V1** + *g*22**I2**.

1. 8 W
2. 8∠45° VA
3. 16 W
4. 16∠45° VA
5. 8∠-45° VA

**Solution:** With **V2** = 0, **I2** = -*g*21**V1**/*g*22 = -**V1**; substituting in the first equation, **I1** = **V1**(*g*11 – *g*12) = 2**V1**; **S** = .

**Version 1:** *Vm* = 2 V; **S** = 8 W

**Version 2:** *Vm* = 3 V; **S** = 18 W

**Version 3:** *Vm* = 4 V; **S** = 32 W

**Version 4:** *Vm* = 5 V; **S** = 50 W

**Version 5:** *Vm* = 6 V; **S** = 72 W.

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**13.** Two circuits are cascaded, with isolation.

The first circuit, of open-circuit transfer function *H*1(*s*), is a first-order, normalized, lowpass Butterworth filter of passband gain of 2. The overall open-circuit transfer function *H*(*s*) = *VO*(*s*)/*VI*(*s*) is that of a second-order, normalized, lowpass Butterworth response of passband gain of 1/2. (a) Derive the open-circuit transfer function *H*2(*s*) and specify its zeros and poles (3 grades); (b) sketch the magnitude Bode plots of *H*1(*jω*) and *H*(*jω*) (3 grades); (c) determine |*H*2(*jω*)| and evaluate |*H*2(*j*1)| in dB (3 grades); (d) derive the equations of the low-frequency and high-frequency asymptotes of |*H*2(*jω*)| and evaluate the 3-dB cutoff frequency (4 grades).

**Solution:** (a) ; ; it has a zero at *s* = -1, in addition to the zero at infinity, and a pair of complex conjugate poles at *s* = .

(b)  , low-pass gain = 20log10(2) = 6 dB, 3-dB cutoff frequency is 1 rad/s,

slope of high-frequency asymptote is -20 dB/decade; ; low-pass gain = 20log10(1/2) = -6 dB, 3-dB cutoff frequency is 1 rad/s, slope of high-frequency asymptote is -40 dB/decade.

(c) , |*H*2(*j*1)| = 20log10(1/4) = -12 dB.

(d) the low-frequency asymptote as *ω* → 0 is 20log10(1/4) = -12 dB; the high-frequency asymptote as *ω* → ∞ is -12 dB – 20log10*ω*, the 3-dB cutoff frequency is given by: , or 1 +  = 2 + 2 , or  – 2 – 1 = 0, which gives , or *ωc* = 1.55 rad/s.

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**14.** The switch is opened at *t* = 0 after being closed for a long time. (a) Represent the circuit in the *s* domain for

*t* ≥ 0+ (5 grades) ; (b) derive *VC*(*s*) (4 grades); (c) determine *vC*(*t*), *t* ≥ 0+ (4 grades).

**Solution:** (a) The initial voltage across *C* is 32 V and the initial current through *L* is 1 A. The circuit in the *s* domain is

as shown.

(b) ;  =

==

= .

(c) To obtain *vC*(*t*), *VC*(*s*) is expressed as: *VC*(*s*) = =  . Hence, *vC*(*t*) = u(*t*) V, *t* is in ms, or, *vC*(*t*) = *u*(*t*) V.

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**15.** (a) Reflect the circuit on the primary (source) side to the secondary

(load) side and determine the *z* parameters of the two-port

circuit between the source and the load (6 grades);

(b) determine *ZL* for maximum power transfer to

*ZL* (4 grades) and calculate this power (3 grades). The z-parameter equations are: *V*1 = *z*11*I*1 + *z*12*I*2; *V*2 = *z*21*I*1 + *z*22*I*2.

**Solution:** (a) the effective turns ratio from the primary side to the secondary side is 1.5. The source becomes 15 V and impedances are multiplied by (1.5)2 = 2.25 to become 9 Ω and – *j*9 Ω. The circuit on the secondary side becomes as shown. The circuit is symmetric, so that *z*12 = *z*21 = -*j*9 Ω, and *z*11 = *z*22 = 9 – *j*9 = 9(1 – *j*) Ω.

(b) *VTh* at the load terminals is 

; *ZTh* = 9 +  9 +  Ω; *ZLm* = 13.5 + *j*4.5 Ω; the power transferred to the load is  W.

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**16.** *f*(*t*) consists of three pulses of unit amplitude centered

at the origin, each pulse being of duration *T*/2 and

separated from the adjacent pulse by *T*/2. It is required

to derive the convolution function *y*(*t*) = *f*(*t*)\*cos using the convolution-in-time property of the Fourier transform.

**Solution:** The FT of the middle pulse is . Adding a delayed pulse and an advanced pulse gives:  = . Multiplying by the FT of cos: . This product is non-zero only at . The term  is even in *ω* and evaluates at  to . The product of the two FTs becomes: = . This is the FT of *y*(*t*). It follows that *y*(*t*) = . Check: direct application of the convolution integral gives the same result:  +  + .